Answer on Question #40467, Differential Calculus, Equations

Find the integral surface of the linear partial differential equation
$x(y^2+z)p - y(x^2+z)q = (x^2-y^2)z$, which contains the straight line $x+y=0$ , $z=1$ .

**Solution.**

Given

$$x(y^2+z)p - y(x^2+z)q = (x^2-y^2)z \qquad (1)$$

Lagrange's auxiliary equations of (1) are

$$\frac{dx}{x(y^2+z)} = \frac{dy}{-y(x^2+z)} = \frac{dz}{(x^2-y^2)z}$$

thus, the solution is the system of equations:

$$\begin{cases} xyz = c_1 \\ x^2+y^2-2z = c_2 \end{cases}$$

Taking $t$ as a parameter, the given equation of the dtraight line $x+y=0$ , $z=1$ can be put in parametric form $x=t$ , $y=-t$ , $z=1$.

Using this,

$$\begin{cases} xyz = c_1 \\ x^2+y^2-2z = c_2 \end{cases}$$

may be re-written as

$$\begin{cases} -t^2 = c_1 \\ 2t^2-2 = c_2 \end{cases}$$

Eliminating $t$ from the equations , we have

$$\begin{cases} 2(-c_1)-2 = c_2 \\ 2c_1+c_2+2 = 0 \end{cases}$$

Putting values of $c_1$ and $c_2$, the desired integral surface is

$$2xyz + x^2 + y^2 - 2z + 2 = 0$$

**Answer:** $2xyz + x^2 + y^2 - 2z + 2 = 0$