

Answer on Question #40451, Math, Statistics and Probability

41 republicans, 2 Independent, 57 Democrats. If 3 senators are selected at random, what is the probability that they will all be republicans? What is the possibility they will all be Democrats? What is the probability that there will be 1 from each party, including the Independent?

Solution

C_k^n - the number of k-combinations from a given set of n elements.

$$C_k^n = \frac{n!}{(n-k)!k!} = \frac{n(n-1)(n-2) \cdots (n-k+1)}{k!},$$

where n! denotes the factorial of n.

Number of ways to select 3 senators from total $N = 41 + 2 + 57 = 100$:

$$n(S) = C_3^{100} = 161700.$$

Number of ways to select 3 republicans from 41:

$$n(R) = C_3^{41} = 10660.$$

The probability that they will all be republicans

$$P(R) = \frac{n(R)}{n(S)} = \frac{10660}{161700} = 0.066.$$

Number of ways to select 3 democrats from 57:

$$n(D) = C_3^{57} = 29260.$$

The probability that they will all be democrats

$$P(D) = \frac{n(D)}{n(S)} = \frac{29260}{161700} = 0.18.$$

Number of ways to select 1 from each party, including the Independent:

$$n(DRI) = C_1^2 C_1^{57} \cdot C_1^{41} = 2 \cdot 57 \cdot 41 = 4674.$$

The probability that there will be 1 from each party:

$$P(DRI) = \frac{n(DRI)}{n(S)} = \frac{4674}{161700} = 0.029.$$