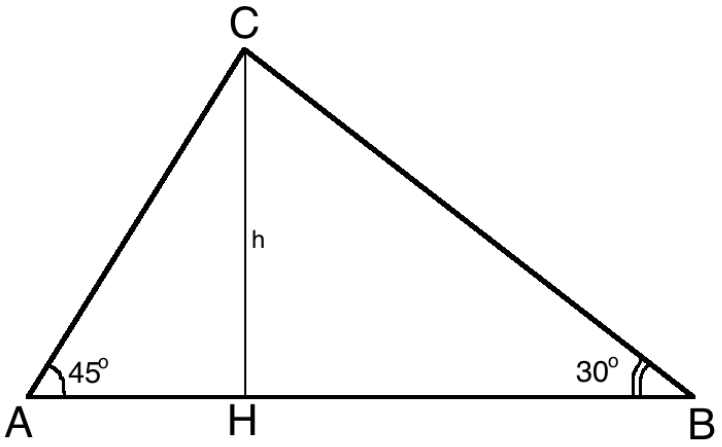
Answer on Question#40444 – Math – Geometry

From two points 240m apart on a horizontal road running east from a mountain the angles of elevation of its top are, respectively $45°$ and $30°$. How high above the road is from the top of the mountain.

Solution:

$\mathrm{A} = \text{point at } 45 \text{ degrees}$
$\mathrm{B} = \text{point at } 30 \text{ degrees}$
$\mathrm{C} = \text{point at top of mountain}$
$\mathrm{AB} = 240\mathrm{m} - \text{length of the horizontal road}$



Firstly, we need to find $\angle \mathrm{C}$.
So

$$\angle \mathrm{C} = 180° - 45° - 30° = 105°$$

Using sine rule:

$$\frac{\mathrm{AB}}{\sin(105°)} = \frac{\mathrm{BC}}{\sin(45°)}$$

Thus,

$$\mathrm{BC} = \mathrm{AB} \cdot \frac{\sin 45°}{\sin 105°} \qquad (1)$$

From the right triangle BCH:

$$\sin 30° = \frac{\mathrm{h}}{\mathrm{BC}} \Rightarrow \mathrm{h} = \mathrm{BC} \sin 30° \qquad (2)$$

$(1)\text{in}(2)$:

$$\mathrm{h} = \mathrm{AB} \cdot \frac{\sin 45°}{\sin 105°} \sin 30° = 240\mathrm{m} \cdot \frac{\sin 45°}{\sin 105°} \sin 30° = 87.85\mathrm{m}$$

Answer: top of the mountain is $87.85\mathrm{m}$ above the road.