Answer on Question #40422 – Math - Combinatorics | Number Theory

Show that k|v is necessary to form a (v, k, lenda)-design.

Solution.

Let's start with a definition.

A *balanced incomplete block design* (BIBD) with parameters (v, k, λ) is an ordered pair $(V, \mathcal{B})$ where V is a set of v objects called *points*, $\mathcal{B}$ is a collection of not necessarily distinct k-subsets of V called *blocks*, and every pair of distinct points are contained in exactly λ blocks. We can say simply "**$(\boldsymbol{v}, \boldsymbol{k}, \boldsymbol{\lambda})$-design**".

So if there exists a (v, k, λ)-design, then

1. $k(k-1) \mid \lambda v(v-1)$,
2. $(k-1) \mid \lambda(v-1)$.

These conditions are necessary for the existence of a BIBD with parameters (v, k, λ). A good way to see this is to consider the contrapositive of the statement: *if either* (1) *or* (2) *is false, then a* (v, k, λ)*-design does not exist*. The conditions are not sufficient: it is false that if v, k and λ are such that $(k-1) \mid \lambda(v-1)$ and $k(k-1) \mid \lambda v(v-1)$, then a (v, k, λ)-design exists.

So the condition $k \mid v$ is necessary for the existence of a resolvable (v, k, λ)-design.