Answer on Question #40414 – Math – Analytic Geometry

If a,b,c be three unit vector sch that ax(bxc)=b/2. Find the angle which a makes with b & c , B & c being non- parallel.

Solution.

Let angles between $\vec{a}$ and $\vec{b}$ and between $\vec{a}$ and $\vec{c}$ be α and β respectively.

We have,

$$\vec{a} \times (\vec{b} \times \vec{c}) = \frac{\vec{b}}{2}$$

By property $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c}) \cdot \vec{b} - (\vec{a} \cdot \vec{b}) \cdot \vec{c}$:

$$(\vec{a} \cdot \vec{c}) \cdot \vec{b} - (\vec{a} \cdot \vec{b}) \cdot \vec{c} = \frac{\vec{b}}{2}$$

$$\left(\vec{a} \cdot \vec{c} - \frac{1}{2}\right) \cdot \vec{b} - (\vec{a} \cdot \vec{b}) \cdot \vec{c} = 0$$

or

$$\vec{a} \cdot \vec{c} = \frac{1}{2} \quad \text{and} \quad \vec{a} \cdot \vec{b} = 0 \qquad (\text{As } \vec{b} \text{ and } \vec{c} \text{ are non} - \text{paralell})$$

Then the angles can be found:

$$\cos\beta = \frac{1}{2}, \quad \cos\alpha = 0$$

or

$$\beta = \frac{\pi}{3}, \quad \alpha = \frac{\pi}{2}.$$

Answer: angle between $\vec{a}$ and $\vec{b}$ is $\pi/3$, angle between $\vec{a}$ and $\vec{c}$ is $\pi/2$.