**Answer on Question #40389 – Math - Differential Calculus**

We have $c = (\ell, m, o)$ and the radius $r = 1$. The equation is

$$(x-\ell)^2 + (y-m)^2 + (z-o)^2 - 1^2 = 0$$

Let $\ell$ be a parameter. Then we have

$$F(x, y, z, \ell) = 0.$$

The envelope of the family of spheres is determined from the equations

$$F(x, y, z, \ell) = 0, \qquad \frac{\partial F}{\partial \ell}(x, y, z, \ell) = 0.$$

Since

$$\frac{\partial F}{\partial \ell} = -2(x-\ell) = 0,$$

we find $\ell = x$.

The envelope of the family of spheres is given by

$$F[x, y, z, \ell(x)] = (y-m)^2 + (z-o)^2 - 1 = 0.$$

This is the equation of a circular cylinder of radius $1$ whose axis coincides with the $x$-axis.

Consider another case. Let $m$ be a parameter. Similarly

$$\frac{\partial F}{\partial m} = -2(y-m) = 0,$$

then

$$m = y.$$

The envelope of the family of spheres is given by

$$F[x, y, z, m(y)] = (x-\ell)^2 + (z-o)^2 - 1 = 0.$$

This is the equation of a circular cylinder of radius $1$ whose axis coincides with the $y$-axis.

And the third case. Let $o$ be a parameter. Similarly

$$\frac{\partial F}{\partial o} = -2(z-o) = 0,$$

then

$$o = z.$$

The envelope of the family of spheres is given by

$$F[x, y, z, o(z)] = (x-\ell)^2 + (y-m)^2 - 1 = 0.$$

This is the equation of a circular cylinder of radius $1$ whose axis coincides with the $z$-axis.

So we have three equations, that give us the envelope of the family of spheres.

**Answer:**

$$(y-m)^2 + (z-o)^2 - 1 = 0,$$
$$(x-\ell)^2 + (z-o)^2 - 1 = 0,$$
$$(x-\ell)^2 + (y-m)^2 - 1 = 0.$$