Answer on Question #40388 – Math - Differential Calculus

Find the differential equation of the space curve in which the two families of surfaces ax2+by2+cz2=u and a2x2+b2y2+c2z2=v intersect.

Solution.

We have the two families of surfaces:

$$u(x,y,z) = ax^2 + by^2 + cz^2 = 0, \;\; v(x,y,z) = a^2x^2 + b^2y^2 + c^2z^2 = 0$$

Let's find the differential equations of the space curve. Form the matrix of partial derivatives:

$$\begin{pmatrix} u'_x & u'_y & u'_z \\ v'_x & v'_y & v'_z \end{pmatrix} = \begin{pmatrix} 2ax & 2by & 2cz \\ 2a^2x & 2b^2y & 2c^2z \end{pmatrix}.$$

Then find determinants of the matrix:

$$\begin{vmatrix} u'_x & u'_y \\ v'_x & v'_y \end{vmatrix} = \begin{vmatrix} 2ax & 2by \\ 2a^2x & 2b^2y \end{vmatrix} = 4ab^2xy - 4a^2bxy = 4abxy(b-a) = f_1$$

$$\begin{vmatrix} u'_y & u'_z \\ v'_y & v'_z \end{vmatrix} = \begin{vmatrix} 2by & 2cz \\ 2b^2y & 2c^2z \end{vmatrix} = 4bc^2yz - 4b^2cyz = 4bcyz(c-b) = f_2$$

$$\begin{vmatrix} u'_z & u'_x \\ v'_z & v'_x \end{vmatrix} = \begin{vmatrix} 2cz & 2ax \\ 2c^2z & 2a^2x \end{vmatrix} = 4ca^2zx - 4c^2azx = 4cazx(a-c) = f_3$$

So, we have the differential equations:

$$\begin{cases} 4abxy(b-a) = f_1 \\ 4bcyz(c-b) = f_2 \\ 4cazx(a-c) = f_3 \end{cases}$$

that completely describe the space curve.