Answer on Question #40387, Math, Differential Calculus

Solve the differential equation:

$xdy-(3y+x^5y^{1/3})dx=0$

Solution:

This equation is called the Bernoulli's equation.

Rewrite this equation

$xy'-3y=x^5y^{1/3}$

Divide both sides by $\frac{3}{2}xy^{1/3}$

$$\frac{2y'}{3y^{\frac{1}{3}}}-\frac{2y^{\frac{2}{3}}}{x}=\frac{2}{3}x^4$$

Let $z(x)=y^{\frac{2}{3}}$, which gives $\frac{dz}{dx}=\frac{\frac{2}{3}y'}{y^{\frac{1}{3}}}$. Then we have equation:

$$\frac{dz}{dx}-2\frac{z}{x}=\frac{2}{3}x^4$$

Let $\mu(x)=e^{\int -\frac{2}{x}dx}=\frac{1}{x^2}$

Multiply both sides by $\mu(x)$

$$\frac{1}{x^2}\frac{dz}{dx}-2\frac{1}{x^2}\frac{z}{x}=\frac{2}{3}\frac{1}{x^2}x^4=\frac{2}{3}x^2$$

Apply the reverse product rule $g\frac{df}{dx}+f\frac{dg}{dx}=\frac{d}{dx}(fg)$ on the left hand side:

$$\frac{d}{dx}\left(\frac{z}{x^2}\right)=\frac{2x^2}{3}$$

Integrate both sides with respect to x:

$$\int\frac{d}{dx}\left(\frac{z}{x^2}\right)dx=\int\frac{2x^2}{3}dx$$

$\frac{z}{x^2}=\frac{2x^3}{9}+C$, where C is an arbitrary constant.

$$z=x^2\left(\frac{2x^3}{9}+C\right)$$

$$y=z^{\frac{3}{2}}=\left(x^2\left(\frac{2x^3}{9}+C\right)\right)^{\frac{3}{2}}$$

Answer:

$$y=\left(x^2\left(\frac{2x^3}{9}+C\right)\right)^{\frac{3}{2}}$$