Answer on Question#40384 – Math – Differential Calculus

Solve the differential equation:

$$y\sin(2x)\,dx = (1 + y^2 + \cos(2x))dy.$$

Solution:

We have

$$y\sin(2x)\,dx - (1 + y^2 + \cos(2x))dy = 0,$$

$$M(x,y)dx + N(x,y)dy = 0,$$

where

$$M(x,y) = y\sin(2x)\,, \qquad N(x,y) = -(1 + y^2 + \cos(2x)).$$

Thus

$$\frac{\partial M}{\partial y} = \sin(2x)\,, \frac{\partial N}{\partial x} = 2\sin(2x).$$

Because $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$ then we can try to find an integrating factor $m(x,y)$.

We get

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{-M(x,y)} = \frac{\sin(2x) - 2\sin(2x)}{-y\sin(2x)} = \frac{-\sin(2x)}{-y\sin(2x)} = \frac{1}{y}.$$

Because

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{-M(x,y)}$$

is the function of variable y only then we have next differential equation

$$\frac{1}{m(y)}\frac{dm}{dy} = \frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{-M(x,y)},$$

$$\frac{dm}{m(y)} = \frac{dy}{y},$$

$$\int \frac{dm}{m(y)} = \int \frac{dy}{y},$$

$$\ln m(y) = \ln y,$$

$$\boxed{m(y) = y}$$

Thus we get next differential equation

$$(y\sin(2x)\,dx-(1+y^2+\cos(2x))dy)\cdot m(y)=0,$$

$$y^2\sin(2x)\,dx-(y+y^3+y\cos(2x))dy=0,$$

$$M_1(x,y)dx+N_1(x,y)dy=0,$$

where

$$M_1(x,y)=y^2\sin(2x)\,,\qquad N_1(x,y)=-(y+y^3+y\cos(2x)).$$

Thus

$$\frac{\partial M_1}{\partial y}=2y\sin(2x)\,,\frac{\partial N_1}{\partial x}=2y\sin(2x).$$

Because $\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$ then we get differential equation in exact differentials. Thus we have

$$u(x,y)=\int M_1(x,y)dx=\int y^2\sin(2x)\,dx=-\frac{y^2}{2}\cos(2x)+\varphi(y).$$

To find function $\varphi(y)$ we get next equation

$$\frac{\partial}{\partial y}\left(-\frac{y^2}{2}\cos(2x)+\varphi(y)\right)=-(y+y^3+y\cos(2x)),$$

$$-y\cos(2x)+\frac{d\varphi}{dy}=-y-y^3-y\cos(2x),$$

$$\frac{d\varphi}{dy}=-y-y^3,$$

$$\varphi(y)=\int(-y-y^3)dy=-\frac{y^2}{2}-\frac{y^4}{4}+c.$$

So

$$u(x,y)=-\frac{y^2}{2}\cos(2x)-\frac{y^2}{2}-\frac{y^4}{4}+c.$$

Thus the general solution of the differential equation is

$$-\frac{y^2}{2}\cos(2x)-\frac{y^2}{2}-\frac{y^4}{4}+c=0,$$

$$-\frac{y^2}{2}\cos(2x)-\frac{y^2}{2}-\frac{y^4}{4}=-c,$$

$$\frac{y^2}{2}(1+\cos(2x))+\frac{y^4}{4}=c,$$

$$\boxed{2y^2(1+\cos(2x))+y^4=C}$$

where $C=4c$.

Answer:

$$\boxed{2y^2(1+\cos(2x))+y^4=C}$$