Answer on Question #40379, Math, Linear Algebra

If { V_1, V_2, V_3 } is a linearly independent set in R^3 , then so is { $V_1 + V_2 - 2V_3$, $V_1 - 2V_2 + V_3$, $-2V_1 + V_2 + V_3$ }.

Solution.

It's False.

Since { $V1, V2, V3$ } is linearly independent, the only way to write the zero vector as a linear combination of V_1, V_2, V_3 is

$$0\boldsymbol{V_1} + 0\boldsymbol{V_2} + 0\boldsymbol{V_3} = \boldsymbol{0}$$

Consider writing the zero vector as a linear combination of { $V_1 + V_2 - 2V_3$, $V_1 - 2V_2 + V_3$, $-2V_1 + V_2 + V_3$ }. That is, what c_1, c_2, c_3 satisfy

$$c_1(V_1 + V_2 - 2V_3) + c_2(V_1 - 2V_2 + V_3) + c_3(-2V_1 + V_2 + V_3) = \boldsymbol{0}$$

$$V_1(c_1 + c_2 - 2c_3) + V_2(c_1 - 2c_2 + c_3) + V_3(-2c_1 + c_2 + c_3) = 0$$

Since the set { $V1, V2, V3$ } is linearly independent, we know that

$$\begin{cases} c_1 + c_2 - 2c_3 = 0 \\ c_1 - 2c_2 + c_3 = 0 \\ -2c_1 + c_2 + c_3 = 0 \end{cases}$$

Solution of this equation is $c_2 = c_1, c_3 = c_1$

I can put $c_1 = 1$, so that $c_2 = 1$ and $c_3 = 1$. Which means, that exist the value of coefficients which is not equal to 0. But this contradicts our assumption.

Hence, the set is { $V_1 + V_2 - 2V_3$, $V_1 - 2V_2 + V_3$, $-2V_1 + V_2 + V_3$ } is not linearly independent.

Answer: the set is not linearly independent.