Answer on Question #40372 – Math – Calculus

A friend says that finding differentials is as easy as finding derivatives – you just multiply the derivative by dx. Is your friend right?

Solution.

Consider the example. Let $f(x) = x^2$. Let's find a derivative:

$$\frac{df}{dx} = \frac{d}{dx}(x^2) = 2x$$

and differential:

$$df = d(x^2) = 2xdx$$

So if we multiply the derivative by dx, we receive the differential.

Consider another example. Let we have function of two variables:

$$f(x, y) = xy$$

Let's find the derivative with respect to x:

$$\frac{df}{dx} = \frac{\partial f}{\partial x} + \frac{\partial f}{\partial y}\frac{dy}{dx} = y + x\frac{dy}{dx},$$

with respect to y:

$$\frac{df}{dy} = \frac{\partial f}{\partial x}\frac{dx}{dy} + \frac{\partial f}{\partial y} = y\frac{dx}{dy} + x$$

and differential:

$$df = d(xy) = xdy + ydx$$

or

$$df = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy = ydx + xdy$$

If we multiply the derivative with respect to x (or y) by dx (or dy) we receive:

$$\frac{df}{dx} \cdot dx = df = ydx + xdy \quad \left(\text{or } \frac{df}{dy} \cdot dy = df = ydx + xdy\right)$$

So we receive our differential.

These examples show that **the friend is right**.