

The population of a city is expected to be

$$P(x) = x(x^2 + 36)^{-1/2}$$

million people after x years. Find the average population between year $x=0$ and year $x=8$.

Solution:

The average population is

$$\begin{aligned} a &= \frac{1}{8-0} \int_0^8 P(x) dx = \frac{1}{8} \int_0^8 x(x^2 + 36)^{-1/2} dx = \frac{1}{8} \int_0^8 \frac{(x^2 + 36)^{-1/2}}{2} d(x^2 + 36) = \\ &= \frac{1}{16} \cdot \frac{(x^2 + 36)^{-1/2+1}}{-1/2+1} \Big|_0^8 = \frac{1}{8} \cdot (x^2 + 36)^{1/2} \Big|_0^8 = \frac{1}{8} \cdot \left((8^2 + 36)^{1/2} - (0^2 + 36)^{1/2} \right) = \\ &= \frac{1}{8} \cdot \left((64 + 36)^{1/2} - (0 + 36)^{1/2} \right) = \frac{1}{8} \cdot \left((100)^{1/2} - (36)^{1/2} \right) = \\ &= \frac{1}{8} \cdot \left((100)^{1/2} - (36)^{1/2} \right) = \frac{1}{8} \cdot (10 - 6) = \frac{1}{2} \text{ (million people)} \end{aligned}$$

Answer:

$$\frac{1}{2} \text{ (million people)}$$