Answer on Question #40201, Math, Statistics and Probability

A box contains 5white and 7 black balls. Two successive drawn of 3 balls are made.

1) with replacement

2) without replacement

The probability that the 1st draw would produce white balls and 2nd draw would produce black balls are

Solution

 C_k^n - the number of k-combinations from a given set of n elements.

$$C_k^n = \frac{n!}{(n-k)!\,k!} = \frac{n(n-1)(n-2)\cdots(n-k+1)}{k!},$$

where n! denotes the factorial of n.

1) When the balls are replaced before 2nd draw

Total balls in the box = 5+7=12.

3 balls can be drawn out of 12 balls in C_3^{12} ways.

3 white balls can be drawn out of 5 white balls is C_3^5 ways.

The probability of drawing 3 white balls

$$P(3W) = \frac{C_3^5}{C_3^{12}}.$$

3 balls can be drawn out of 12 balls in C_3^{12} ways.

3 black balls can be drawn out of 7 white balls is C_3^7 ways.

The probability of drawing 3 black balls

$$P(3B) = \frac{C_3^7}{C_3^{12}}$$

Since both the events are dependent, the required probability is

$$P(3W \text{ and } 3B) = \frac{C_3^5}{C_3^{12}} \cdot \frac{C_3^7}{C_3^{12}} = \frac{10}{220} \cdot \frac{35}{220} = 0.007.$$

2) When the balls are not replaced before 2nd draw

Total balls in the box = 5+7=12.

3 balls can be drawn out of 12 balls in C_3^{12} ways.

3 white balls can be drawn out of 5 white balls is C_3^5 ways.

The probability of drawing 3 white balls

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$$P(3W) = \frac{C_3^5}{C_3^{12}}.$$

After the first draw, balls left are 9.

3 balls can be drawn out of 9 balls in C_3^9 ways.

3 black balls can be drawn out of 7 white balls is \mathcal{C}_3^7 ways.

The probability of drawing 3 black balls

$$P(3B) = \frac{C_3^7}{C_3^9}.$$

Since both the events are dependent, the required probability is

$$P(3W \text{ and } 3B) = \frac{C_3^5}{C_3^{12}} \cdot \frac{C_3^7}{C_3^9} = \frac{10}{220} \cdot \frac{35}{84} = 0.019.$$