Answer on Question#40159 - Math – Linear Algebra:

Let $(V, <, >)$ be an inner product space over $\mathbb{C}$ and T belongs to (V). Prove that if

$\forall\, x, y \in V: \; < Tx\,, Ty > = < x, y >$, then T is unitary.

Solution.

We need to prove that $T^*T = I$, where T^* is an adjoin operator, and I is an identity operator.

$$\forall x, y \in V: (x, y) = (Tx, Ty) = (x, T^*Ty) \Rightarrow \forall x, y \in V: (x, T^*Ty - y) = 0 \Rightarrow$$

$$\Rightarrow \forall y \in V: T^*Ty - y = 0 \Rightarrow \forall y \in V: T^*Ty = y \Rightarrow T^*T = I.$$