Answer on Question#40157, Math, Linear Algebra

Find the dual basis of the basis $e_1 = (1,1,2)$, $e_2 = (1,0,1)$, $e_3 = (2,1,0)$ of the vector space R^3 over R.

Solution.

We need to find vectors:

$$e_1' = (x_1, y_1, z_1), e_2' = (x_2, y_2, z_2), e_3' = (x_3, y_3, z_3)$$

The dual base vectors should satisfy this:

$$e_i e^j = \delta_{ij}$$

The condition equals to 3 systems of 3 equations in 3 unknowns. Solving each system will give you one vector for the dual base. Here goes the first one which would be for the first dual base vector:

$$\begin{cases} x_1 + y_1 + 2z_1 = 1 \\ x_1 + z_1 = 0 \\ 2x_1 + y_1 = 0 \end{cases}$$

$$x_1 = -\frac{1}{3}, y_1 = \frac{2}{3}, z_1 = \frac{1}{3}$$

for second vector:

$$\begin{cases} x_2 + y_2 + 2z_2 = 0 \\ x_2 + z_2 = 1 \\ 2x_2 + y_2 = 0 \end{cases}$$

$$x_2 = \frac{2}{3}, y_2 = -\frac{4}{3}, z_2 = \frac{1}{3}$$

for third vector:

$$\begin{cases} x_3 + y_3 + 2z_3 = 0 \\ x_3 + z_3 = 0 \\ 2x_3 + y_3 = 1 \end{cases}$$

$$x_3 = \frac{1}{3}, y_3 = \frac{1}{3}, z_3 = -\frac{1}{3}$$

Answer: $e_1' = \left(-\frac{1}{3}, \frac{2}{3}, \frac{1}{3}\right)$, $e_2' = \left(\frac{2}{3}, -\frac{4}{3}, \frac{1}{3}\right)$, $e_3' = \left(\frac{1}{3}, \frac{1}{3}, -\frac{1}{3}\right)$.