Answer on Question #40024, Math, Linear Algebra

Find the normal canonical form of the quadratic form 2xy+2yz-x2-y2-z2. Hence, compute its signature.

Solution.

We should use **Lagrange's Reduction**. The reduction of a quadratic form to canonical form can be carried out by a procedure known as Lagrange's Reduction, which consists essentially of repeated completing of the square.

$$Q = 2xy + 2yz - x^2 - y^2 - z^2 =$$

rewrite it

$$= -x^2 + 2xy - y^2 + 2yz - z^2 =$$

First we should complete the square of terms with x:

$$= -(x^2 - 2xy + y^2) - z^2 + 2yz = -(x-y)^2 - z^2 + 2yz = -(x-y)^2 + Q_1$$

Do it with Q_1:

$$Q_1 = -z^2 + 2yz = -z^2 + 2yz - y^2 + y^2 = -(z^2 - 2yz + y^2) + y^2 = -(z-y)^2 + y^2$$

Therefore,

$$Q = -(x-y)^2 + y^2 - (z-y)^2$$

Inspection of this last expression for Q shows those substitutions that will reduce Q to the canonical form :

$$x' = x - y$$

$$y' = y$$

$$z' = z - y$$

Substituting x', y' and z' into the last expression for Q gives

$$Q = -x'^2 + y'^2 - z'^2$$

which is of the canonical form , where Q is expressed in terms of the new variables x', y' and z' .

The signature of the quadratic form is the number p of positive squared terms in the reduced form.

So the signature of Q is 1.

Answer:

$$Q = -x'^2 + y'^2 - z'^2$$

$$Signature = 1$$