## Answer on Question \#40120, Math, Linear Algebra

Find out basis vector for $\mathrm{V}=$ matrix of $2 \times 2$ of complex numbers $(\mathrm{C})$ over the field of real numbers $(\mathrm{R})$. What is dimension of matrix of $2 \times 2$ of complex numbers $(C)$ over real numbers $(R)$ ?

## Solution

The basis of the vector space

$$
\operatorname{Mat}_{2}(\mathbb{R})=\left(\begin{array}{ll}
a_{11} & a_{12} \\
a_{21} & a_{22}
\end{array}\right)
$$

over $\mathbb{R}$ is

$$
e=\left\{\left(\begin{array}{ll}
1 & 0 \\
0 & 0
\end{array}\right),\left(\begin{array}{ll}
0 & 0 \\
1 & 0
\end{array}\right),\left(\begin{array}{ll}
0 & 1 \\
0 & 0
\end{array}\right),\left(\begin{array}{ll}
0 & 0 \\
0 & 1
\end{array}\right)\right\} .
$$

We want elements such that you can write matrix of the form

$$
\operatorname{Mat}_{2}(\mathbb{C})=\left(\begin{array}{ll}
a_{11}+b_{11} i & a_{12}+b_{12} i \\
a_{21}+b_{21} i & a_{22}+b_{22} i
\end{array}\right)
$$

But we can only multiply the elements with real numbers. Therefore we need two elements for every position in the matrix. One element takes care of the complex part, one element for the real part.

So basis of the vector space $\operatorname{Mat}_{2}(\mathbb{C})=\left(\begin{array}{ll}a_{11}+b_{11} i & a_{12}+b_{12} i \\ a_{21}+b_{21} i & a_{22}+b_{22} i\end{array}\right)$ over $\mathbb{R}$ is

$$
e_{c}=\left\{\left(\begin{array}{ll}
1 & 0 \\
0 & 0
\end{array}\right),\left(\begin{array}{ll}
0 & 0 \\
1 & 0
\end{array}\right),\left(\begin{array}{ll}
0 & 1 \\
0 & 0
\end{array}\right),\left(\begin{array}{ll}
0 & 0 \\
0 & 1
\end{array}\right),\left(\begin{array}{ll}
i & 0 \\
0 & 0
\end{array}\right),\left(\begin{array}{ll}
0 & 0 \\
i & 0
\end{array}\right),\left(\begin{array}{ll}
0 & i \\
0 & 0
\end{array}\right),\left(\begin{array}{ll}
0 & 0 \\
0 & i
\end{array}\right)\right\}
$$

The dimension of matrix of $2 \times 2$ of real numbers $(R)$ over real numbers $(R)$ is 4 . The change from the real numbers to the complex numbers has the effect of doubling the dimensions of the transformation. Indeed, a $2 \times 2$ complex matrix has eight "parameters" not four. That's why dimension of matrix of $2 \times 2$ of complex numbers(C) over real numbers( $R$ ) is 8 .

