

Answer on Question #40054 – Math - Other

Assignment

Show that the curvilinear coordinate system defined by the following equations is orthogonal:

$$x=uv\cos(\alpha)$$

$$y=uv\sin(\alpha)$$

$$z=1/2(u^2-v^2)$$

note here  $\alpha$  stands for alpha

Solution

$$\vec{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} uv\cos(\alpha) \\ uv\sin(\alpha) \\ \frac{1}{2}(u^2 - v^2) \end{pmatrix}$$

Derivatives of the radius vector:

$$\vec{r}_u = \begin{pmatrix} x_u \\ y_u \\ z_u \end{pmatrix} = \begin{pmatrix} v\cos(\alpha) \\ v\sin(\alpha) \\ u \end{pmatrix}; \vec{r}_v = \begin{pmatrix} x_v \\ y_v \\ z_v \end{pmatrix} = \begin{pmatrix} u\cos(\alpha) \\ u\sin(\alpha) \\ -v \end{pmatrix}; \vec{r}_\alpha = \begin{pmatrix} x_\alpha \\ y_\alpha \\ z_\alpha \end{pmatrix} = \begin{pmatrix} -uv\sin(\alpha) \\ uv\cos(\alpha) \\ 0 \end{pmatrix};$$

Scalar products:

$$\vec{r}_u \cdot \vec{r}_v = uv\cos^2(\alpha) + uv\sin^2(\alpha) - uv = 0$$

$$\vec{r}_u \cdot \vec{r}_\alpha = -uv^2\cos(\alpha)\sin(\alpha) + uv^2\cos(\alpha)\sin(\alpha) = 0$$

$$\vec{r}_v \cdot \vec{r}_\alpha = -u^2v\cos(\alpha)\sin(\alpha) + u^2v\cos(\alpha)\sin(\alpha) = 0$$

It means that  $\vec{r}_u, \vec{r}_v$  and  $\vec{r}_\alpha$  can be chosen as a basis and these vectors are orthogonal.