Answer on Question#40032 – Math – Differential Calculus| Equation

Solve the differential equation $\boldsymbol{y''} = \mathbf{1} + (\boldsymbol{y'})^{\mathbf{2}}$.

Solution.

Let $v(x) = y'(x) \rightarrow y'' = v'$ and from our equation:

$$v' = v^2 + 1 \rightarrow \frac{v'}{v^2+1} = 1 \rightarrow \int \frac{v'}{v^2+1} dx = \int 1\, dx \rightarrow \int \frac{1}{v^2+1} dv = \int 1\, dx \rightarrow$$

$$\rightarrow \arctan(v) = x + c_1 \rightarrow v = \tan(x + c_1).$$

Then: $y' = v = \tan(x + c_1) \rightarrow y = \int \tan(x + c_1)\, dx \rightarrow y = -\ln(\cos(x + c_1) + c_2.$

Answer: $\boldsymbol{y(x) = -\ln(\cos(x + c_1) + c_2}$, where $\boldsymbol{c_1}$ and $\boldsymbol{c_2}$ are the arbitrary constants.