Answer on Question #40027, Math, Differential Calculus

According to Newton's law of cooling, the rate at which a substance cools in moving air is proportional to the difference between the temperature of the substance and that of the air. If the temperature of the air is 290 degree centigrade and the substance cools from 370 to 330 degree centigrade in 10 minutes, find when the temperature will be 295 degree centigrade.

Solution.

Let T be the temperature of the substance at the time t. Then, by hypothesis, we have

$$\frac{dT}{dt} = -\lambda(T-290)$$

$$\frac{dT}{T-290} = -\lambda dt$$

where λ is positive constant of proportionality.

Integrating our equation between the limits $t = 0$, $T = 370\ K$ and $= 10\ min$, $T = 330\ K$, we have

$$\int_{370}^{330} \frac{dT}{T-290} = -\lambda \int_{0}^{10} dt$$

$$\ln(T-290)\ |_{370}^{330} = -10\lambda$$

$$-10\lambda = \ln(330-290) - \ln(370-290) = \ln 40 - \ln 80 = \ln\frac{40}{80} = \ln\frac{1}{2} = -\ln 2$$

$$\lambda = \frac{ln2}{10}$$

Again, assuming that $t = t'$ minutes when $T = 295K$ and so integrating the equation between the limits $= 0$, $T = 370K\ and\ t = t', T = 295K$, we have

$$\int_{370}^{295} \frac{dT}{T-290} = -\lambda \int_{0}^{t\prime} dt$$

$$ln5 - ln80 = \ln\frac{5}{80} = -\ln 16 = -\lambda t'$$

$$\lambda t' = \ln 16 = 4\ln 2$$

Therefore,

$$\begin{cases} \lambda t' = 4\ ln\ 2 \\ \lambda = ln2/10 \end{cases}$$

$$t' = \frac{4\ln 2}{ln2} \cdot 10 = 40\ minutes.$$

Answer: $t' = 40\ minutes$