Answer on Question #40026 – Math - Differential Calculus

We have the equation $y'(x) + P(x)y(x) - Q(x) = 0$. Show that the set of real (or complex) situations of this equation does not form a real (or complex) vector space.

Solution.

Let y_1 and y_2 be any two solutions of the given differential equation. Then it is to be shown that $y_1 + y_2$ and ay_1 are also solutions where a is a scalar. We have

$$(y_1 + y_2)' + P(x)(y_1 + y_2) - Q(x) = y_1' + y_2' + P(x)y_1 + P(x)y_2 - Q(x)$$
$$= \big(y_1' + P(x)y_1 - Q(x)\big) + \big(y_2' + P(x)y_2 - Q(x)\big) + Q(x) = Q(x) \neq 0,$$

as y_1 and y_2 are solutions.

Hence $(y_1 + y_2)$ is not a solution.

Further,

$$(ay_1)' + P(x)(ay_1) - Q(x) = ay_1' + aP(x)y_1 - aQ(x) + aQ(x) - Q(x) = Q(x)(a - 1) \neq 0,$$

as y_1 is a solution.

Thus ay_1 is not a solution.

So we see that our set does not form a real (or complex) vector space.