

Answer on Question #40021, Math, Linear Algebra

Let a quadratic form has expression  $6x^2+5xy+10y^2$  with respect to the standard basis of  $R^2$ . Find its expression with respect to the new basis  $\{(3,1), (-2,1)\}$  of  $R^2$ .

Solution.

$$Q = 6x^2 + 5xy + 10y^2$$

Standard basis in  $R^2$  is:

$$\left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\}$$

New basis:

$$\left\{ \begin{pmatrix} 3 \\ 1 \end{pmatrix}, \begin{pmatrix} -2 \\ 1 \end{pmatrix} \right\}$$

So to write  $Q$  with respect to new basis we should go to another terms:

$$x = 3x' + y'$$

$$y = -2x' + y'$$

Therefore,

$$\begin{aligned} Q' &= 6(3x' + y')^2 + 5(3x' + y')(-2x' + y') + 10(-2x' + y')^2 = \\ &= 6(9x'^2 + 6x'y' + y'^2) + 5(-6x'^2 + x'y' + y'^2) + 10(4x'^2 - 4x'y' + y'^2) = \\ &= 54x'^2 + 36x'y' + 6y'^2 - 30x'^2 + 5x'y' + 5y'^2 + 40x'^2 - 40x'y' + 10y'^2 = \\ &= 64x'^2 + x'y' + 21y'^2 = Q' \end{aligned}$$

So  $Q'$  is a quadratic form in new basis  $\left\{ \begin{pmatrix} 3 \\ 1 \end{pmatrix}, \begin{pmatrix} -2 \\ 1 \end{pmatrix} \right\}$ .

Answer:  $Q' = 64x'^2 + x'y' + 21y'^2$