Answer on question #40005 – Math – Matrix

If A and B are two matrices of same order and rank (A) = rank (B) = n, then rank(A+B)=n , for n>=1 (T/F)

Solution

Suppose we get two matrices $n \times n$ with rank n

$$A = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{pmatrix} \ and \ B = \begin{pmatrix} b_{11} & \cdots & b_{1n} \\ \vdots & \ddots & \vdots \\ b_{n1} & \cdots & b_{nn} \end{pmatrix}$$

This means that rows in these matrices are independent. But if exist such λ that

$$a_{ik} = \lambda b_{jk}, \qquad b_{ik} = \lambda a_{jk}, \qquad for\ some\ i\ and\ j\ belong\ to\ [1; n]\ and\ all\ k = \overline{1; n}$$

Then for the sum of this matrices we get

$$a_{ik} + b_{ik} = \lambda(a_{jk} + b_{jk}), \qquad \forall k \in [1; n].$$

That it mean that A+B has at least dependent rows and its rank less than n.

For example, let

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}, \qquad B = \begin{pmatrix} 3 & 4 \\ 1 & 2 \end{pmatrix}$$

Both matrices have rank 2, but tier sum

$$A + B = \begin{pmatrix} 1+3 & 2+4 \\ 3+1 & 4+2 \end{pmatrix} = \begin{pmatrix} 4 & 6 \\ 4 & 6 \end{pmatrix}$$

Has rank 1.

Answer: False.