Answer on Question #39804 – Math – Calculus

An ice cube tray filled with tap water is placed in the freezer and the temperature of the water is changing at the rate of -12e^-0.2t degree fahrenheit per hour after t hours. The original temperature of the tap water was 70 degrees.

Questions:

a. Find a formula for the temperature of the water that has been in the freezer for t hours.

b. When will be the ice be ready? (Water freezed at 32 degrees)

Solution.

We have

$$f(t) = -12e^{-0.2t}$$

a. We can find the formula for the temperature of the water that has been in the freezer for t hours by taking the integral:

$$F(t) = \int f(t)dt = \int (-12e^{-0.2t})dt = -12\int e^{-0.2t}dt = \frac{-12}{-0.2}e^{-0.2t} + C = 60e^{-0.2t} + C$$

We have a condition: the original temperature of the tap water was 70 degrees.

Then

$$F(0) = 60 \cdot e^{-0.2 \cdot 0} + C = 70$$

$$60 + C = 70$$

$$C = 10$$

So

$$F(t) = 60e^{-0.2t} + 10$$

b. The ice be ready at 32 degrees:

$$F(t) = 32 = 60e^{-0.2t} + 10$$

$$60e^{-0.2t} = 22$$

$$e^{-0.2t} = \frac{22}{60} = \frac{11}{30}$$

$$-0.2t = \ln\frac{11}{30}$$

$$t = \ln\frac{11}{30} \cdot \left(-\frac{1}{0.2}\right) = -5 \cdot \ln\frac{11}{30} \approx 5$$

Answer:

a. $60e^{-0.2t} + 10$;

b. In about 5 hours.