Answer on Question #39798 – Math – Calculus

A piece of wire 20 m long is cut into two pieces. One piece is bent into a square and the other is bent into an equilateral triangle. (Give your answers correct to two decimal places.)
(a) How much wire should be used for the square in order to maximize the total area?
(b) How much wire should be used for the square in order to minimize the total area?

Solution:
Let $\boldsymbol{x}$ be a piece of wire for a square.
Let $\boldsymbol{y}$ be a piece of wire for an equilateral triangle.
$\left(\frac{x}{4}\right)^2$ – the area of the square,
x – perimeter of the square;
$y^2 \cdot \frac{\sqrt{3}}{4}$ – the area of an equilateral triangle,
y – perimeter of an equilateral triangle.

From the problem above,
$x + y = 20$. (1)

The total area
$S = \left(\frac{x}{4}\right)^2 + y^2 \cdot \frac{\sqrt{3}}{4}$. (2)

By (1), $S = \left(\frac{x}{4}\right)^2 + y^2 \cdot \frac{\sqrt{3}}{4} = \frac{x^2}{16} + (20 - x)^2 \cdot \frac{\sqrt{3}}{4}$.
To maximize or minimize the total area, let take a derivative
$\dot{S} = \frac{x}{8} - \frac{\sqrt{3}(20-x)}{2}$.
Let $\dot{S} = 0$, whence
$\frac{x}{8} - \frac{\sqrt{3}(20-x)}{2} = 0$,
$x = 4\sqrt{3}(20 - x)$,
$x(1 + 4\sqrt{3}) = 80\sqrt{3}$,
$x = \frac{80\sqrt{3}}{1+4\sqrt{3}} \approx 17,47736 \approx 17,48$.
$\ddot{S} = \frac{1}{8} + \frac{\sqrt{3}}{2} > 0$ for all x.
So, $x = 17,48$ is minimum of S.
So, $x = 20$ is maximum of S.

Answer: (a) 20; **(b)** 17,48.