Answer on Question#39765, Math, Calculus

An ice cube tray filled with tap water is placed in the freezer and the temperature of the water is changing at the rate of -12e^-0.2t degree Fahrenheit per hour after t hours. The original temperature of the tap water was 70 degrees.

Questions:

a. Find a formula for the temperature of the water that has been in the freezer for t hours.

b. When will be the ice be ready? (Water freezed at 32 degrees)

Solution

The rate of change of the temperature of the water:

$$\frac{df}{dt} = -12e^{-0.2t}.$$

Let's find a formula for the temperature of the water that has been in the freezer for t hours by taking the integral:

$$f(t) = \int \frac{df}{dt}dt = \int (-12e^{-0.2t})\,dt = -12\int e^{-0.2t}\,dt = \frac{-12}{-0.2}e^{-0.2t} + C = 60e^{-0.2t} + C.$$

We know that original temperature of the tap water was 70 degrees:

$$f(0) = 60e^{-0.2\cdot 0} + C = 70 \to C = 10.$$

So

$$f(t) = 60e^{-0.2t} + 10.$$

The ice is ready at 32 degrees:

$$f(t_{ice}) = 60e^{-0.2t_{ice}} + 10 = 32 \to e^{-0.2t_{ice}} = \frac{32-10}{60} = \frac{11}{30}.$$

Then

$$t_{ice} = -\frac{1}{0.2}\ln\frac{11}{30} = -5\ln\frac{11}{30} \approx 5 \text{ hours}.$$

Answer: a. $60e^{-0.2t} + 10$; b. 5 hours.