Answer on Question#39721, Math, Statistics and Probability

The following data represent the travel time (in minutes) to school. Algebra course. Treat the nine students as a population

Amanda 39 Scot 45 Amber 21 Tim 9 Erica 11 Tiffany 12 Nicole 30 Mike 32 Glenn 39

(a) Determine the population standard deviation.

(b) Find three simple random samples of size 4, and determine the sample standard deviation of each sample.

(c) Which samples underestimate the population standard deviation? Which overestimate the population standard deviation?

Solution

(a) Determine the population standard deviation.

The population standard deviation σ, is obtained by taking the square root of the population variance. That is,

$$\sigma = \sqrt{\sigma^2}.$$

$$\sigma^2 = \frac{\sum(x_i - \mu)^2}{N}.$$

$\mu = \frac{\sum x_i}{N}$ – Population mean.

In our case:

$$\mu = \frac{39 + 45 + 21 + 9 + 11 + 12 + 30 + 32 + 39}{9} = 26.4.$$

$$\sigma^2 = \frac{(39 - 26.4)^2 + (45 - 26.4)^2 + (21 - 26.4)^2 + (9 - 26.4)^2 + (11 - 26.4)^2 + (12 - 26.4)^2 + (30 - 26.4)^2 + (32 - 26.4)^2 + (39 - 26.4)^2}{9}.$$

$$\sigma^2 = 164.9,$$

$$\sigma = \sqrt{164.9} = 12.8.$$

Answer: 12.8.

(b) Find three simple random samples of size 4, and determine the sample standard deviation of each sample.

The sample standard deviation, s, is obtained by taking the square root of the sample variance. That is,

$$s = \sqrt{s^2}.$$

$$s^2 = \frac{\sum(x_i - \bar{x})^2}{n - 1}.$$

n - sample size, $\bar{x}$ – sample mean.

First sample: Amanda 39 Scot 45 Amber 21 Tim 9.

$$\bar{x} = \frac{39 + 45 + 21 + 9}{4} = 28.5.$$

$$s^2 = \frac{(39 - 28.5)^2 + (45 - 28.5)^2 + (21 - 28.5)^2 + (9 - 28.5)^2}{4 - 1} = 273.$$

The sample standard deviation $s = \sqrt{273} = 16.5$.

Second sample: Erica 11 Tiffany 12 Nicole 30 Mike 32.

$$\bar{x} = \frac{11 + 12 + 30 + 32}{4} = 21.25.$$

$$s^2 = \frac{(11 - 21.25)^2 + (12 - 21.25)^2 + (30 - 21.25)^2 + (32 - 21.25)^2}{4 - 1} = 127.6.$$

The sample standard deviation $s = \sqrt{127.6} = 11.3$.

Third sample: Amanda 39 Amber 21 Erica 11 Glenn 39.

$$\bar{x} = \frac{39 + 21 + 11 + 39}{4} = 27.5.$$

$$s^2 = \frac{(39 - 27.5)^2 + (21 - 27.5)^2 + (11 - 27.5)^2 + (39 - 27.5)^2}{4 - 1} = 193.$$

The sample standard deviation $s = \sqrt{193} = 13.9$.

(c) *Which samples underestimate the population standard deviation? Which overestimate the population standard deviation?*

Second sample ($s = 11.3$) underestimate the population standard deviation ($s < \sigma$).

First ($s = 16.5$) and Third ($s = 13.9$) samples overestimate the population standard deviation ($s > \sigma$).