Answer on question 39627 – Math – Calculus

Differentiate from first principles $F(x) = 0.5 + \sqrt{x}$.

Solution

Let us consider the arbitrary points $P(x; y)$ and $Q(x + \Delta x; y + \Delta y)$ so that $y = F(x), y + \Delta y = F(x + \Delta x)$. Then the slope of the secant is

$$\frac{\Delta y}{\Delta x} = \frac{F(x + \Delta x) - F(x)}{\Delta x}.$$

The slope of the tangent is

$$F'(x) = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{F(x + \Delta x) - F(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{0.5 + \sqrt{x + \Delta x} - 0.5 - \sqrt{x}}{\Delta x} =$$

$$= \lim_{\Delta x \to 0} \frac{\sqrt{x + \Delta x} - \sqrt{x}}{\Delta x} = \lim_{\Delta x \to 0} \frac{\left(\sqrt{x + \Delta x} - \sqrt{x}\right)\left(\sqrt{x + \Delta x} + \sqrt{x}\right)}{\Delta x\left(\sqrt{x + \Delta x} - \sqrt{x}\right)} = \lim_{\Delta x \to 0} \frac{x + \Delta x - x}{\Delta x\left(\sqrt{x + \Delta x} - \sqrt{x}\right)} =$$

$$= \lim_{\Delta x \to 0} \frac{1}{\left(\sqrt{x + \Delta x} - \sqrt{x}\right)} = \frac{1}{2\sqrt{x}}.$$

Answer: $F'(x) = \frac{1}{2\sqrt{x}}$.