Answer on the Question #39540 – Math – Statistics and Probability

In a factory turning out razor blade, there is a small chance of 1/500 for any blade to be defective. The blades are supplied in a packet of 10. Use Poisson distribution to calculate the approximate number of packets containing blades with no defective, one defective, two defectives and three defectives in a consignment of 10,000 packets.

Solution.

Number of defective blades in a packet has binomial distribution $B(n, p)$ with parameters $n = 10$ and $p = \frac{1}{500} = 0.002$

Binomial distribution can be approximated using Poisson distribution with parameter $a = np = 10 * 0.002 = 0.02$.

We should calculate the number of defective blades in a packet. Let ξ equals to number of the defective blades. $\xi = 0,1,2,3$.

Using the Poisson formula $p_m = P(\xi = m) = \frac{a^m}{m!} e^{-a}$.

Hence we have:

$$p_0 = e^{-0.002} \approx 0.9802$$

Using that $p_{m+1} = p_m \frac{a}{m+1}$ $(from\ the\ Poisson\ formula)$ we have

$$p_1 = p_0 \frac{0.02}{1} = 0.0196$$

$$p_2 = p_1 \frac{0.02}{2} = 0.000196$$

$$p_3 = p_2 \frac{0.02}{3} \approx 0$$

Thus expected numbers of packets with no, defective, 1 defective, 2 defective and 3 defective blades are:

$$n_0 = 10000 p_0 \approx 9802$$

$$n_1 = 10000 p_1 \approx 196$$

$$n_2 = 10000 p_2 \approx 2$$

$$n_3 \approx 0$$