Answer on Question#39518 – Math – Other

Solve the initial value problem: (d^2x/dt^2)-3(dx/dt)-10x=0, x(0)=1 and x'(0)=0.

Solution:

Rewrite our equation as $\ddot{x} - 3\dot{x} - 10x = 0$, where x=x(t) be function with the argument t. We have linear homogeneous differential equation of second order.
The respective characteristic equation $k^2 - 3k - 10 = 0$.

Discriminant of this quadratic equation is $D = 9 - 4(-10) = 9 + 40 = 49$, than $k_1 = \frac{3+7}{2} = 5, k_2 = \frac{3-7}{2} = -2$ are the roots of quadratic equation.

By the method of Euler, we have the solution of differential equation $x(t) = c_1e^{5t} + c_2e^{-2t}$, where c_1 and c_2 are the constants.

Let find c_1 and c_2, and solve the initial value problem.

Let x(0)=1. Than

$$1 = c_1e^0 + c_2e^0 = c_1 + c_2. \quad (1)$$

Let x'(0)=0. Than

$$\dot{x}(t) = 5c_1e^{5t} - 2c_2e^{-2t},$$

whence $0 = \boldsymbol{5c_1e}^0 - \boldsymbol{2c_2e}^0$ and

$$\boldsymbol{5c_1 = 2c_2}. \quad (2)$$

Than, by (1) and (2)

$$c_1 = \frac{2}{15}, c_2 = \frac{1}{3}.$$

Answer: $x(t) = \frac{\mathbf{2}}{\mathbf{15}}e^{5t} + \frac{1}{3}e^{-2t}$.