Answer on question #39464 - Math - Algebra

$a \cdot \sin\sqrt{x} + b \cdot \sin(x)\cos(x) + c \cdot \cos\sqrt{x}$. Find range of solution of x

Solution:

We have to solve given equation $a\sin\sqrt{x} + b\sin(x)\cos(x) + c\cos\sqrt{x}$ according to x.

There are no restrictions on the domain of sine and cosine functions, therefore their domain is such that x ∈ R. It should be note, that the range for both $y = \sin(x)$ and $y = \cos(x)$ is between -1 and 1. Different transformations of these functions in the form of shifts and stretches will affect the range but not the domain.

Based on trigonometric formulas can replace.

$$\sin(2x) = 2\sin(x)\cos(x)$$

$$\sin(x)\cos(x) = \frac{\sin(2x)}{2}$$

Substitute into given function. We can rewrite given equation:

$$a \cdot \sin\sqrt{x} + b \cdot \frac{\sin(2x)}{2} + c \cdot \cos\sqrt{x} = a \cdot \sin\sqrt{x} + \frac{b}{2}\sin(2x) + c \cdot \cos\sqrt{x}$$

Since in this equation, the value of members and, in, and to be identified and marked, we can assert that the domain could not be determined unambiguously. Therefore, we can cite the following value. We carry replacement trigonometric functions.

$$c \cdot \cos\sqrt{x} = \frac{c}{\csc(\frac{\pi}{2} + \sqrt{x})}$$

$$a \cdot \sin\sqrt{x} = \frac{a}{\csc\sqrt{x}}$$

$$b \cdot \frac{\sin(2x)}{2} = \frac{0.5b}{\csc(2x)}$$

Now we can write.

$$a \cdot \sin\sqrt{x} + \frac{1}{2}\sin(2x) + c \cdot \cos\sqrt{x} = \frac{a}{\csc\sqrt{x}} + \frac{0.5b}{\csc(2x)} + \frac{c}{\csc\left(\frac{\pi}{2} + \sqrt{x}\right)}$$

So, finally we can write the domain of the equation:

Domain: $x \in R, \csc\sqrt{x} \neq 0, \csc(2x) \neq 0, \sec\sqrt{x} \neq 0$ and $x \geq 0, -\frac{\pi}{2} < \pi n - \sqrt{x} < 0$ and $2x > \pi n, 2x < \pi(n+1), n \in Z$.

Range: The range is all the possible values of y.

We also can graphically represent this equation. Graph is shown below.