Answer on Question #3946 – Math – Statistics and Probability

Question

A GE light bulb is supposed to last for 1200 hours. In fact, light bulbs of this type last only 1185 hours with a standard deviation of 70 hours. What is the probability that a sample of light bulbs will have an average life of at least 1200 hours?

Solution

Assume that the population is normal:

$$X_i \sim N(\mu, \sigma^2) = N(1185, 70^2).$$

Then the distribution of the sample mean

$$\bar{X} = \frac{X_1 + X_2 + X_3 + \cdots + X_{100}}{100}$$

is exactly normal with mean

$$\mu_{\bar{x}} = E(\bar{X}) = \mu = 1185 \text{ hours}$$

and standard deviation

$$\sigma_{\bar{x}} = sd(\bar{X}) = \frac{\sigma}{\sqrt{n}} = \frac{70}{\sqrt{100}} = 7 \text{ hours}.$$

The standardized variable

$$Z = \frac{\bar{X} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{\bar{X} - 1185}{7}$$

Is distributed as $N(0,1)$.

The following value z of Z corresponds to the value $\bar{x} = 1200$ of $\bar{X}$:

$$z = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{1200 - 1185}{7} = 2.142857.$$

Therefore,

$$P(\bar{X} \geq 1200) = P\left(\frac{\bar{X} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} \geq \frac{1200 - \mu_{\bar{x}}}{\sigma_{\bar{x}}}\right) = P\left(Z \geq \frac{1200 - 1185}{7}\right) = P(Z \geq 2.142857) =$$
$$= 1 - P(Z < 2.142857) = 1 - 0.983938 = 0.016062,$$

because using the command

$$= \text{NORM. S. DIST}(2{,}142857; \text{TRUE})$$

from Microsoft Excel we can see that

$$z = 2.142857$$

gives

$$P(Z < 2.142857) = 0.983938.$$

Only rarely, just over one time in a hundred tries of 100 light bulbs, would the average life exceed 1200 hours.

Answer: 0.016062.