Answer on Question#39429 - Math - Analytic Geometry

A soccer ball has 12 pentagons and 20 hexagons, making a perfect ball shape; I'm told that any other combination of polygon that also form a ball shape MUST have 'at least' 12 pentagons, yet I have a photo of a ball shaped with close to 90 hexagons and possibly ONLY 6 pentagons. I will email the photo if you wish to work out the exact numbers ... then tell me if you can how it can work with only 6 pentagons?

Solution

Soccer ball graph is a planar, 3-regular and 3-connected graph, the faces of which are only pentagons and hexagons. Let the number of vertices, edges, pentagons, and hexagons of a soccer ball graph G be denoted by v, e, f_5, and f_6, respectively. It is easy to see that

$3v=2e$ and $5f_5+6f_6=2e$.

Euler's formula:

$v+(f_5+f_6)-e=2$ or $6v+6f_5+6f_6=12+4e+2e$. From last formula $f_5=12$. Thus the number of pentagons in a soccer ball graph is exactly 12.

Answer

It can not work with only 6 pentagons.