Answer on Question #39341 – Math – Linear Algebra

Question. Find the point B in $\mathbb{R}^3$ in the plane $p : x + 2y - 3z = 12$ closest to the point $A = (1, 1, 1)$.

Solution. The point B in the plane p closest to A is a unique point on p such that the vector $\overrightarrow{AB}$ is orthogonal to p. Thus to solve the problem we should find the intersection point of p with the line l orthogonal to p and passing through A.

The assumption that $\overrightarrow{AB}$ is orthogonal to p is equivalent to the assumption that $\overrightarrow{AB}$ is parallel to the normal vector $\vec{n}$ to p.

Coordinates of $\vec{n}$ are the coefficients of the left hand side of the equation of $p : x + 2y - 3z = 12$. Thus

$$\vec{n} = (1, 2, -3).$$

Therefore the parametric equation of the line l orthogonal to p and passing through A is the following one:

$$\begin{cases} x = 1 + t, \\ y = 1 + 2t, \\ z = 1 - 3t. \end{cases}$$

Substituting these formulas into the equation of p we get:

$$1 + t + 2(1 + 2t) - 3(1 - 3t) = 12,$$
$$1 + t + 2 + 4t - 3 + 9t = 12,$$
$$14t = 12,$$
$$t = \frac{12}{14} = \frac{6}{7}.$$

Hence point B has the following coordinates:

$$B = \left(1 + \frac{6}{7},\ 1 + 2 \cdot \frac{6}{7},\ 1 - 3 \cdot \frac{6}{7}\right) = \left(\frac{13}{7},\ \frac{19}{7},\ -\frac{11}{7}\right).$$

Answer.

$$B = \left(\frac{13}{7},\ \frac{19}{7},\ -\frac{11}{7}\right).$$