Answer on Question #39182 – Math – Integral Calculus

1. Evaluate the integral:

$$\frac{1}{T}\int_0^{T/2}\sin\left(\frac{2\pi}{T}t\right)e^{-jn\frac{2\pi}{T}t}\,dt.$$

Solution.

Let use the formula $\sin a=\dfrac{e^{ja}-e^{-ja}}{2j}$.

$$I=\frac{1}{T}\int_0^{T/2}\sin\left(\frac{2\pi}{T}t\right)e^{-jn\frac{2\pi}{T}t}\,dt=\frac{1}{T}\int_0^{T/2}\frac{e^{j\frac{2\pi}{T}t}-e^{-j\frac{2\pi}{T}t}}{2j}e^{-jn\frac{2\pi}{T}t}\,dt=\frac{1}{2Tj}\int_0^{T/2}\left[e^{-j(n-1)\frac{2\pi}{T}t}-e^{-j(n+1)\frac{2\pi}{T}t}\right]dt=$$

$$=\frac{1}{2Tj}\cdot\left[-\frac{e^{-j(n-1)\frac{2\pi}{T}t}}{j(n-1)\frac{2\pi}{T}}+\frac{e^{-j(n+1)\frac{2\pi}{T}t}}{j(n+1)\frac{2\pi}{T}}\right]\Bigg|_0^{T/2}=\frac{1}{2Tj}\cdot\left[-\frac{e^{-j(n-1)\pi}-1}{j(n-1)\frac{2\pi}{T}}+\frac{e^{-j(n+1)\pi}-1}{j(n+1)\frac{2\pi}{T}}\right]=$$

$$=\frac{1}{4\pi}\left[\frac{e^{-j(n-1)\pi}-1}{n-1}-\frac{e^{-j(n+1)\pi}-1}{n+1}\right].$$

If $n\in Z$, than $e^{-j(n-1)\pi}=(-1)^{n-1}$, $e^{-j(n+1)\pi}=(-1)^{n+1}=(-1)^{n-1}$, so

$$I=\frac{1}{4\pi}\left[\frac{(-1)^{n-1}-1}{n-1}-\frac{(-1)^{n-1}-1}{n+1}\right]=\frac{(-1)^{n-1}-1}{4\pi}\left(\frac{1}{n-1}-\frac{1}{n+1}\right)=\frac{(-1)^{n-1}-1}{2(n^2-1)\pi}=\begin{cases}0, & n=2k-1\\ -\dfrac{1}{(n^2-1)\pi}, & n=2k\end{cases}.$$

Answer: $\dfrac{1}{4\pi}\left[\dfrac{e^{-j(n-1)\pi}-1}{n-1}-\dfrac{e^{-j(n+1)\pi}-1}{n+1}\right].$