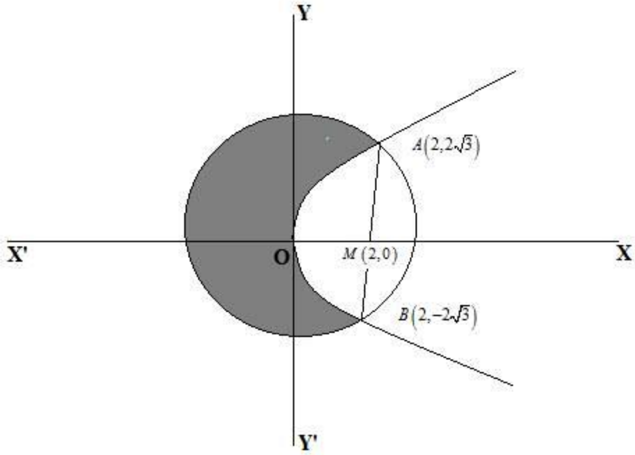
Answer on Question#39220 – Math – Other

using integral find the area of that part of circle x2+y2=16, which is exterior to the parabola y2=6x.

Solution:

We have to find the area of that part of circle x2+y2=16, which is exterior to the parabola y2=6x. So, we have to find the shaded region area.



Firstly, we will find the point of intersection,

$$x^2 + 6x = 16$$
$$x^2 + 6x - 16 = 0$$

$$(x + 8)(x - 2) = 0 \Longrightarrow$$
$$x = -8; 2$$

So,

$$y^2 = 6x(-8) = -48 \Longrightarrow y = -\sqrt{48}$$

And

$$y^2 = 6x(2) = 12 \Longrightarrow y = \sqrt{12} \Longrightarrow y = \pm 2\sqrt{3}$$

So, the point of intersection are

$$\left(2, 2\sqrt{3}\right), \left(2, -2\sqrt{3}\right) \text{and } \left(-8, -\sqrt{48}\,\right)$$

We will take only first and second point because third point has imaginary value.

Required area = area of circle – area of the part which is not shaded

So,

$$\text{Required area} = \pi(4)^2 - 2\int_0^2 \sqrt{6x}dx - 2\int_2^4 \sqrt{16 - x^2}\,dx \quad (1)$$

$$\int_0^2 \sqrt{6x}dx = \sqrt{6}\left(\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right)\begin{matrix}2\\0\end{matrix} = \frac{2\sqrt{6}}{3}(2)^{\frac{3}{2}} = \frac{8\sqrt{3}}{3}$$

And

$$\int_2^4 \sqrt{16 - x^2}\,dx$$
$$\text{let } x = 4\sin\theta \Longrightarrow dx = 4\cos\theta\,d\theta$$

$$\text{limits are } 2 = 4\sin\theta \Longrightarrow \theta = \frac{\pi}{6} \text{ and } 4 = 4\sin\theta \Longrightarrow \theta = \frac{\pi}{2}$$

$$\Longrightarrow \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sqrt{16 - (4\sin\theta)^2}\, 4\cos\theta\, d\theta = 16 \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (\cos\theta)^2 d\theta = 16 \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{1 - \cos 2\theta}{2} d\theta =$$

$$8\left(\theta - \frac{\sin 2\theta}{2}\right)_{\frac{\pi}{6}}^{\frac{\pi}{2}} = 8\left(\frac{\pi}{2} - \frac{\sin\pi}{2} - \frac{\pi}{6} + \frac{\sin\frac{\pi}{3}}{2}\right) = 8\left(\frac{\pi}{3} + \frac{\sqrt{3}}{4}\right)$$

Now, substitute all values in equation (1), we get,

$$\text{Required area} = \pi(4)^2 - 2\left(\frac{8\sqrt{3}}{3}\right) - 2\left(8\left(\frac{\pi}{3} + \frac{\sqrt{3}}{4}\right)\right)$$

$$= \pi\left(16 - \frac{16}{3}\right) - \frac{16\sqrt{3}}{3} - 4\sqrt{3} =$$

$$= \pi\left(\frac{32}{3}\right) - \frac{28\sqrt{3}}{3} = \frac{4}{3}\left(8\pi - 7\sqrt{3}\right) \text{sq. units}$$

Answer: $\frac{4}{3}\left(8\pi - 7\sqrt{3}\right)$ sq. units.