Answer on Question#39191 - Math – Statistics and Probability

Car security alarms go off at a mean rate of 4.0 per hour in a large Costco parking lot.
Find the probability that in an hour there will be (Round your answers to 4 decimal places.)
   Probability
(a) no alarms
(b) fewer than five alarms
(c) more than seven alarms

Solution

We can assume ξ has a Poisson distrubution. The poisson distribution formula is

$$P(k) = \Pr(\xi = k) = \frac{e^{-\lambda}\lambda^k}{k!}, \quad k=0,1,2,\ldots$$

where P(k) is the probability of the event that a random variable ξ takes on the value k, e is the constant 2.718… , λ (usually written as the Greek letter lambda) is the average number of events

(in our case, λ =4)

a)   k = 0

   $P(0) = \frac{e^{-4}4^0}{0!} = e^{-4} \approx 0.0183$

b) $\xi < 5$

   $P(\xi<5) = |\text{additivity of probability}| = P(\xi =0) + P(\xi =1) + P(\xi =2) + P(\xi =3) + P(\xi =4) =$

   $= \frac{e^{-4}4^0}{0!} + \frac{e^{-4}4^1}{1!} + \frac{e^{-4}4^2}{2!} + \frac{e^{-4}4^3}{3!} + \frac{e^{-4}4^4}{4!} \approx 0.6288$

c) $\xi > 7$

   $P(\xi>7) = |\text{probability of the complementary events}| = 1 - (P(\xi =0) + P(\xi =1) + P(\xi =2) + P(\xi = 3) +$

   $+ P(\xi =4) + P(\xi =5) + P(\xi =6) + P(\xi =7)) = 0.0511$