Answer on Question #39182 – Math – Other

**1.** Show that

$$(\vec{v}\,\vec{\nabla})\,\vec{v} = \frac{1}{2}\vec{\nabla}\, v^2 - \vec{v} \times \left[\vec{\nabla}\times\vec{v}\right]$$

*Proof.*

Let simplify the expressions in Cartesian coordinates.

If $\vec{v} = v_1\,\vec{i} + v_2\,\vec{j} + v_3\,\vec{k}$, than $X$-component of the left-side of the equation:

$$\left(v_1\frac{\partial}{\partial x} + v_2\frac{\partial}{\partial y} + v_3\frac{\partial}{\partial z}\right)v_1 = v_1\frac{\partial v_1}{\partial x} + v_2\frac{\partial v_1}{\partial y} + v_3\frac{\partial v_1}{\partial z}.$$

The vector product:

$$\left[\vec{\nabla}\times\vec{v}\right] = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ v_1 & v_2 & v_3 \end{vmatrix} = \vec{i}\left(\frac{\partial v_3}{\partial y} - \frac{\partial v_2}{\partial z}\right) - \vec{j}\left(\frac{\partial v_3}{\partial x} - \frac{\partial v_1}{\partial z}\right) + \vec{k}\left(\frac{\partial v_2}{\partial x} - \frac{\partial v_1}{\partial y}\right).$$

$X$-component of the right-side of the equation:

$$\frac{1}{2}\frac{\partial}{\partial x}\left(v_1{}^2 + v_2{}^2 + v_3{}^2\right) - \left[v_2\cdot\left(\frac{\partial v_2}{\partial x} - \frac{\partial v_1}{\partial y}\right) + v_3\cdot\left(\frac{\partial v_3}{\partial x} - \frac{\partial v_1}{\partial z}\right)\right] =$$

$$= v_1\frac{\partial v_1}{\partial x} + v_2\frac{\partial v_2}{\partial x} + v_3\frac{\partial v_3}{\partial x} - v_2\frac{\partial v_2}{\partial x} + v_2\frac{\partial v_1}{\partial y} - v_3\frac{\partial v_3}{\partial x} + v_3\frac{\partial v_1}{\partial z} = v_1\frac{\partial v_1}{\partial x} + v_2\frac{\partial v_1}{\partial y} + v_3\frac{\partial v_1}{\partial z}.$$

The $X$- and $Y$- components can be checked in the same way.

**Answer**: The statement is correct.