Answer on Question #39179 - Math – Statistics

Question: The lifetime X of a bulb is a random variable with the probability density function:

$$f(x) = \begin{cases} 6 * (0.25 - (x - 1.5)^2) \ \ when\ 1 \le x \le 2 \\ 0\ otherwise \end{cases}$$

X is measured in multiples of 1000 hrs. What is the probability that none of the three bulbs in a traffic signal have to be replaced in the first 1500 hrs of their operation.

Solution: T_i – the random variable of a lifetime (measured in multiples of 1000 hrs.) of i-th bulb, i = 1, 2, 3. Then, for A = "lifetime of each bulb is longer than 1500",

$$P(A) = P\big((T_1 \ge 1.5) \wedge (T_2 \ge 1.5) \wedge (T_3 \ge 1.5)\big) = P(T_1 \ge 1.5) * P(T_2 \ge 1.5) * P(T_3 \ge 1.5)$$
$$= P(T_1 \ge 1.5)^3.$$

$$P(T_1 \ge 1.5) = \int_{1.5}^{\infty} f(x)dx = \int_{1.5}^{2} 6 * (0.25 - (x - 1.5)^2)dx = \left(\frac{3}{2}x - 2(x - 1.5)^3\right)\Big|_{1.5}^{2}$$
$$= (3 - 2 * 0.5^3) - \left(\frac{9}{4}\right) = 3 - \frac{1}{4} - \frac{9}{4} = 3 - \frac{5}{2} = \frac{1}{2}.$$

Then $P(A) = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$.

Answer: 1/8.