Answer on Question#39178 Math - Other

Evaluate $\iint \vec{A}\,\vec{n}\,dS$, where $\vec{A}=x\cos^2 y\,\vec{i}+xz\,\vec{j}+z\sin^2 y\,\vec{k}$, over the surface of a sphere with its centre at the origin and radius of 4 units.

Solution:

$$\iint \vec{A}\,\vec{n}\,dS=\iiint \operatorname{div}\vec{A}\,dxdydz$$

$$\operatorname{div}\vec{A}=\frac{\partial\left(x\cos^2 y\right)}{\partial x}+\frac{\partial(xz)}{\partial y}+\frac{\partial\left(z\sin^2 y\right)}{\partial z}=\cos^2 y+0+\sin^2 y=1$$

$$\iiint \operatorname{div}\vec{A}\,dxdydz=\iiint 1\cdot dxdydz=\begin{pmatrix} x=r\sin\theta\cos\varphi & z=r\cos\theta \\ y=r\sin\theta\sin\varphi & J=r^2\sin\theta \end{pmatrix}=\int_{\varphi=0}^{2\pi}\int_{\theta=0}^{\pi}\int_{r=0}^{4} r^2\sin\theta\,d\varphi\,d\theta\,dr=$$

$$\int_{\varphi=0}^{2\pi} d\varphi\int_{\theta=0}^{\pi}\sin\theta\,d\theta\int_{r=0}^{4} r^2\,dr=\varphi|_0^{2\pi}\cdot(-\cos\theta)|_0^{\pi}\cdot\left.\frac{r^3}{3}\right|_0^4=2\pi\cdot 2\cdot\frac{4^3}{3}=\frac{256\pi}{3}$$

Answer: $\frac{256\pi}{3}$