Answer on Question#39060 – Math – Integral Calculus

Evaluate: $\int \frac{1}{x\, e^{x}}\ dx.$

Solution.

Integration by parts :

If $u = u(x)$, $v = v(x)$, and the differentials $du = u'(x)\, dx$ and $dv = v'(x)\, dx$, then integration by parts states that

$$\int u(x)v'(x)\,dx = u(x)v(x) - \int u'(x)v(x)\,dx$$

or more compactly:

$$\int u\,dv = uv - \int v\,du.$$

So,

$\int \frac{1}{x\, e^{x}}\ dx = \int \frac{e^{-x}}{x}\ dx = -\int \frac{d(e^{-x})}{x} = -\left(\frac{1}{x * e^{x}} - \int \frac{-1}{x^{2}\, e^{x}}\ dx\right) =$

$= \frac{-1}{x * e^{x}} + \int \frac{1}{x^{2}}\ d(e^{-x}) = \frac{-1}{x * e^{x}} + \frac{1}{x^{2} * e^{x}} - \int \frac{-2}{x^{3}\, e^{x}}\ dx =$

$= \frac{-1}{x * e^{x}} + \frac{1}{x^{2} * e^{x}} - \int \frac{2}{x^{3}}\ d(e^{-x}) =$

$= \frac{-1}{x * e^{x}} + \frac{1}{x^{2} * e^{x}} - \left(\frac{2}{x^{3} * e^{x}} - \int \frac{-2*3}{x^{4}\, e^{x}}\ dx\right) =$

$= \frac{-1}{x * e^{x}} + \frac{1}{x^{2} * e^{x}} - \frac{2}{x^{3} * e^{x}} + \int \frac{6}{x^{4}}\ d(e^{-x}) = \ldots. =$

$$= e^{-x}\left(-\frac{1}{x} + \left(\frac{1}{x}\right)^{2} - \frac{2}{x^{3}} + \frac{6}{x^{4}} - \frac{24}{x^{5}} + O\left(\left(\frac{1}{x}\right)^{6}\right)\right)$$

$$\int \frac{1}{x\, e^{x}}\ dx = Ei(-x) + const,$$

where Ei(x) is the exponential integral.