

Answer on Question#39018 – Math – Trigonometry

If $\tan A = p \tan B$ then prove that $\sin(A+B) = (p+1)(\sin(A-B))/(p-1)$

Solution:

Initial condition:

$$\tan A = p \cdot \tan B$$

We can write the tangent as $\tan \alpha = \frac{\sin \alpha}{\cos \alpha}$:

$$\frac{\sin A}{\cos A} = p \cdot \frac{\sin B}{\cos B}$$

$$p \cos A \cdot \sin B = \sin A \cdot \cos B \quad (1)$$

Sine of the sum and difference:

$$\sin(A + B) = \sin A \cdot \cos B + \cos A \cdot \sin B \quad (2)$$

$$\sin(A - B) = \sin A \cdot \cos B - \cos A \cdot \sin B \quad (3)$$

(1) in (2) and (3):

$$\sin(A + B) = p \cos A \cdot \sin B + \cos A \cdot \sin B = \cos A \cdot \sin B (p + 1) \quad (4)$$

$$\sin(A - B) = p \cos A \cdot \sin B - \cos A \cdot \sin B = \cos A \cdot \sin B (p - 1) \quad (5)$$

From (5):

$$\cos A \cdot \sin B = \frac{\sin(A - B)}{p - 1} \quad (6)$$

(6) in (4):

$$\sin(A + B) = \cos A \cdot \sin B (p + 1) = \frac{(p + 1)(\sin(A - B))}{p - 1}$$