Answer on question 38939 – Math – Differential Calculus

Solve the following IVP, and check the non-characteristic condition first: UU_x + yU_y = x U = 2s on a characteristic curve gamma: x = s, y = s, s element of R.

Solution

$$\begin{cases} uu_x + yu_y = x \\ u = 2s \\ x = y = s \end{cases}$$

First independent integrals are

$$\frac{dx}{u} = \frac{dy}{y} = \frac{du}{x}$$

Therefrom we get

$$\frac{dx}{u} = \frac{du}{x} \;\Rightarrow\; \int x dx = \int u du \;\Rightarrow\; u^2 = x^2 + c_1 \;\Rightarrow\; c_1 = u - x; \qquad (*)$$

$$\frac{dy}{y} = \frac{du}{x} \;\Rightarrow\; \frac{u}{x} = \ln|y| + c_2 \;\Rightarrow\; c_2 = \frac{u}{x} - \ln|y|. \qquad (**)$$

Considering the initial conditions we get

$$c_1 = 2s - s = s, \qquad c_2 = \frac{2s}{s} - \ln|s| = 2 - \ln|s|$$

Or we can write

$$c_2 = 2 - \ln|c_1|$$

Substitute (*) and (**) into this equation

$$\frac{u}{x} - \ln|y| = 2 - \ln|u - x|$$

$$\frac{u}{x} = 2 + \ln\left|\frac{y}{u - x}\right|$$

Answer: $\frac{u}{x} - 2 - \ln\left|\frac{y}{u-x}\right| = 0.$