Answer on Question #38893, Math, Other

The answer to given problem might be obtained using Fourier method. The formula is $u(x,t)=\sum_{n=1}^{\infty}(A_n\cos\pi n\frac{v}{L}x+B_n\sin\pi n\frac{v}{L}t)\sin\pi\frac{n}{L}x$. Using initial conditions, $u(x,0)=\varphi(x), u_t(x,0)=\psi(x)$ it is easy to obtain Fourier coefficients $A_n=\frac{2}{L}\int_0^L\varphi(x)\sin\pi\frac{n}{L}x\,dx$, $B_n=\frac{2}{\pi n v}\int_0^L\psi(x)\sin\pi\frac{n}{L}x\,dx$.

For current problem, $\psi(x)=0$. Thus,

$$A_n=\frac{2}{L}\left(A\int_0^{\frac{L}{4}}x\sin\pi\frac{n}{L}x\,dx+\frac{AL}{4}\int_{\frac{L}{4}}^{\frac{3L}{4}}\sin\pi\frac{n}{L}x\,dx+A\int_{\frac{3L}{4}}^{L}(L-x)\sin\pi\frac{n}{L}x\,dx\right)=$$

$$\frac{2}{L}\left(\frac{AL}{n^2\pi^2}\left(\frac{-1}{4}Ln\pi\cos\pi\frac{n}{4}+L\sin\pi\frac{n}{4}\right)+\frac{AL}{4}\frac{2L\sin\pi\frac{n}{4}\sin\pi\frac{n}{2}}{n\pi}+AL^2\frac{(n\pi\cos3\frac{\pi}{4}n+4\sin3\frac{\pi}{4}n)}{4n^2\pi^2}\right)=$$

$$=\frac{2}{L}\left(\frac{AL^2}{n^2\pi^2}(2\sin\pi\frac{n}{2}\cos\pi\frac{n}{4})\right)=\frac{4AL}{\pi^2n^2}\sin\pi\frac{n}{2}\cos\pi\frac{n}{4} .$$

The solution is $u(x,t)=\sum_{n=1}^{\infty}\frac{4AL}{\pi^2n^2}\sin\pi\frac{n}{2}\cos\pi\frac{n}{4}\cdot\sin\pi\frac{n}{L}x$.