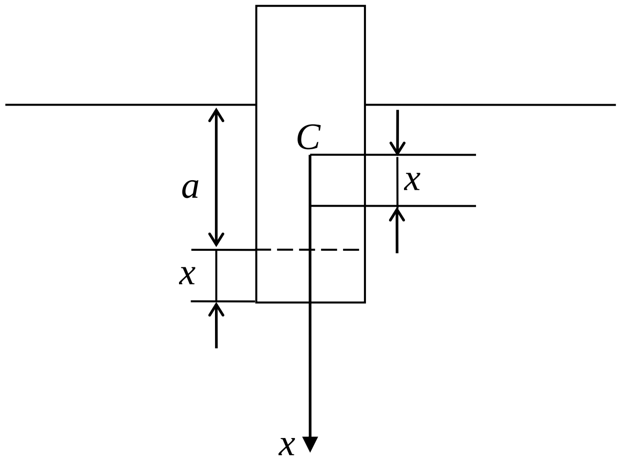
Answer on Question#38888 - Math - Other

A cylinder with a diameter 1.0 m is stands in water with its axis vertical. When depressed slightly and released, it oscillates with a time period of 2 s. Determine the mass of the cylinder.

Solution

We denote the mass of the cylinder by m then its weight is equal to $P = mg$, where $g = 9{,}8\ m/s^2$ is the gravitational acceleration. While being in the position of equilibrium the cylinder is immersed in water to a depth equal to a (See the figure).



In this position the force of gravity and the force of Archimedes are in equilibrium. Let γ denotes the specific weight of the water. Because the volume of the immersed part of the cylinder equals $V = \pi r^2 a$, where the cylinder radius $r = 0{,}5\ m$, the force of Archimedes is equal to $F_{ar} = \gamma V = \pi\gamma r^2 a$. Hence, we have from the equation $P = F_{ar}$, i. e.

$$mg = \pi\gamma r^2 a. \tag{1}$$

To determine the motion of the cylinder with respect to the position of equilibrium we consider the action of water as an additional force of Archimedes. We direct downward the Ox – axis, putting its origin in the placement of equilibrium of the gravity center C.

The force of Archimedes is equal to

$$F_{ar} = \pi\gamma r^2(a + x). \tag{2}$$

From Newton's second law of motion of the center of gravity with using (1) and (2) we have

$$m\ddot{x} = P - F_{ar}$$
$$= mg - \pi\gamma r^2 a - \pi\gamma r^2 x$$
$$= -\pi\gamma r^2 x.$$

Hence, the position $x(t)$ of the mass center satisfies the second-order linear homogeneous differential equation

$$m\ddot{x} + \pi\gamma r^2 x = 0.$$

Dividing the both sides of the equation by $m \neq 0$ we obtain

$$\ddot{x} + \frac{\pi \gamma r^2}{m} x = 0.$$

Now if we denote $k^2 = \pi\gamma r^2/m$ then the last equation is rewritten

$$\ddot{x} + k^2 x = 0. \quad (3)$$

We expect oscillatory motion. If we attempt a solution of (3) of the form $x(t) = A\cos(\omega t + \varphi)$ for some frequency ω and amplitude A, we find upon substitution that $\omega = k$. Therefore the displacement of the cylinder is given by

$$x(t) = A\cos(kt + \varphi).$$

These solutions represent an oscillations of frequency k, and period $T = 2\pi/k$, so $k = 2\pi/T$. Hence

$$m = \frac{\pi \gamma r^2}{k^2} = \frac{\gamma r^2 T^2}{4\pi}. \quad (4)$$

Thus substituting all known values into the formula (4) we evaluate

$$m = \frac{1000 kg/m^3 9{,}8 m/s^2 0{,}25 m^2 4 s^2}{4\pi} \approx 779.8692209\ kg \approx 780\ kg.$$

Answer:

780 *kg*