Answer on Question#38738 – Math - Other

We have the curvilinear coordinate system

$$x = uv\cos\alpha\,,\ \ x = uv\sin\alpha\,,\ \ z = \frac{u^2 - v^2}{2}$$

In curvilinear coordinates, a point in space is specified by the coordinates, and at every such point there is bound a set of basis vectors, which generally are not constant. In orthogonal coordinates the basis vectors vary, they are always orthogonal with respect to each other. In other words,

$$\vec{e}_i \cdot \vec{e}_j = 0 \ \ \text{if} \ \ i \neq j,$$

These basis vectors are by definition the tangent vectors of the curves obtained by varying one coordinate, keeping the others fixed:

$$\vec{e}_i = \frac{\partial \vec{r}}{\partial q_i}$$

where $\vec{r}$ is some point and q_i is the coordinate for which the basis vector is extracted.

We have:

$$\vec{r} = (x, y, z) = \left(uv\cos\alpha\,, uv\sin\alpha\,, \frac{u^2 - v^2}{2}\right)$$

Let's determine the tangent vectors and check their orthogonality:

$$\vec{e}_1 = \frac{\partial \vec{r}}{\partial u} = (v\cos\alpha\,, v\sin\alpha\,, u),$$

$$\vec{e}_2 = \frac{\partial \vec{r}}{\partial v} = (u\cos\alpha\,, u\sin\alpha\,, -v),$$

$$\vec{e}_3 = \frac{\partial \vec{r}}{\partial \alpha} = (-uv\sin\alpha\,, uv\cos\alpha\,, 0).$$

Then

$$\vec{e}_1 \cdot \vec{e}_2 = uv \cdot (\cos\alpha \cdot \cos\alpha + \sin\alpha \cdot \sin\alpha) - uv = uv - uv = 0,$$

$$\vec{e}_2 \cdot \vec{e}_3 = u^2 v \cdot (-\cos\alpha \cdot \sin\alpha + \sin\alpha \cdot \cos\alpha) = u^2 v \cdot 0 = 0,$$

$$\vec{e}_1 \cdot \vec{e}_3 = uv^2 \cdot (-\cos\alpha \cdot \sin\alpha + \sin\alpha \cdot \cos\alpha) = uv^2 \cdot 0 = 0.$$

This shows that all pairs of tangent vectors are orthogonal and therefore that **the coordinate system is an orthogonal one**.