Question#38737, Math, Differential Calculus

Obtain the gradient of the following scalar field :

U(X,Y,Z)=X^2Zcos2Y

Solution

By the definition, $\text{grad}U = \frac{\partial U}{\partial X}\mathbf{i} + \frac{\partial U}{\partial Y}\mathbf{j} + \frac{\partial U}{\partial Z}\mathbf{k}$.

All we have to do then is to find the partial derivatives.

1. If the brackets were omitted, then the partial derivatives are

$$\frac{\partial U}{\partial X} = 2XZ\cos(2Y), \frac{\partial U}{\partial Y} = -2X^2Z\sin(2Y), \frac{\partial U}{\partial Z} = X^2\cos(2Y).$$

Then gradient of the scalar field is the following vector

$$\text{grad}U = 2XZ\cos(2Y)\,\mathbf{i} - 2X^2Z\sin(2Y)\,\mathbf{j} + X^2\cos(2Y)\,\mathbf{k}.$$

Answer

$\text{grad}U = 2XZ\cos(2Y)\,\mathbf{i} - 2X^2Z\sin(2Y)\,\mathbf{j} + X^2\cos(2Y)\,\mathbf{k}$

2. If the brackets were not omitted, then the partial derivatives are equal to

$$\frac{\partial U}{\partial X} = 2XZY\cos 2, \frac{\partial U}{\partial Y} = X^2Z\cos 2, \frac{\partial U}{\partial Z} = X^2Y\cos 2.$$

Then gradient of the scalar field is equal to the following vector

$$\text{grad}U = (2ZY\mathbf{i} + XZ\mathbf{j} + XY\mathbf{k})X\cos 2.$$

Answer

$\text{grad}U = (2ZY\mathbf{i} + XZ\mathbf{j} + XY\mathbf{k})X\cos 2$