Answer on Question#38684 - Math - Other

Question: If the sum of first n even natural numbers is equal to k times the sum of first n odd natural numbers then k =?

a. $\frac{1}{n}$
b. $\frac{n-1}{n}$
c. $\frac{n+1}{2n}$
d. $\frac{n+1}{n}$

Solution. First of all, recall the formula for the sum of n first terms of an arithmetic progression with the first term a_1 and common difference d:

$$S_n = \frac{n}{2}(2a_1 + (n-1)d).$$

Now note that the "**first n even natural numbers**" form an arithmetic progression:

$$2, 4, 6, 8, \ldots$$

with $a_1 = 2$ and $d = 4 - 2 = 2$. We can thus apply the formula above and calculate the sum of these numbers (let us denote it as S_{ev}):

$$S_{ev} = \frac{n}{2}(2 * 2 + (n-1)2) = n(2 + n - 1) = n(n+1).$$

Similarly, the "**first n odd natural numbers**" also form an arithmetic progression:

$$1, 3, 5, 7, \ldots$$

with $a_1 = 1$ and $d = 3 - 1 = 2$. Let us denote their sum as S_{odd}:

$$S_{odd} = \frac{n}{2}(2 * 1 + (n-1)2) = \frac{n}{2}(2 + 2n - 2) = n^2.$$

We are given the condition that the sum of first n even natural numbers is equal to k times the sum of first n odd natural numbers, or

$$S_{ev} = k * S_{odd}.$$

Substitute the obtained values of S_{odd} and S_{ev} into this formula:

$$n(n+1) = kn^2$$

Thus,

$$k = \frac{n(n+1)}{n^2} = \frac{n+1}{n}.$$

Answer. d) $k = \frac{n+1}{n}$.