Answer on Question#38683 - Math - Other

Let S_n denote the sum of n terms of an AP whose first term is a. If the common difference d is given by $d = S_n - kS_{n-1} + S_{n-2}$ then $k = ?$
(a) 1 (b) 2 (c) 3 (d) None

Solution:

The sum of n terms of an AP is: $S_n = \frac{n}{2}(2a + (n-1)d)$

The sum of $(n-1)$terms of an AP is: $S_{n-1} = \frac{n-1}{2}(2a + (n-2)d)$

The sum of $(n-2)$ terms of an AP is: $S_{n-2} = \frac{n-2}{2}(2a + (n-3)d)$

So

$$d = \frac{n}{2}(2a + (n-1)d) - k\frac{n-1}{2}(2a + (n-2)d) + \frac{n-2}{2}(2a + (n-3)d)$$

$$d = a\big(n - k(n-1) + (n-2)\big) + d\left(\frac{n(n-1)}{2} - k\frac{(n-1)(n-2)}{2} + \frac{(n-2)(n-3)}{2}\right)$$

So we have:

$\big(n - k(n-1) + (n-2)\big) = 0$ and

$$\left(\frac{n(n-1)}{2} - k\frac{(n-1)(n-2)}{2} + \frac{(n-2)(n-3)}{2}\right) = 1$$

From the first:

$$k = \frac{2n-2}{n-1} = 2$$

Substituting $k = 2$ into the second:

$$\left(\frac{n(n-1)}{2} - 2 \cdot \frac{(n-1)(n-2)}{2} + \frac{(n-2)(n-3)}{2}\right) = 1$$

Multiplying both sides on 2:

$$n(n-1) - 2(n-1)(n-2) + (n-2)(n-3) = 2$$

Expanding:

$$n^2 - n - 2n^2 + 6n - 4 + n^2 - 5n + 6 = 2$$

$$2 = 2$$

Hence, answer - (b) 2