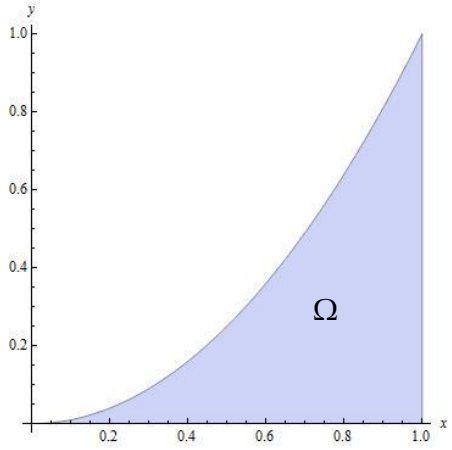
Answer on Question#38682 - Math - Calculus

Question: A plane sheet of material is bound by the curve y = x^2 from x = 0 to x =1, the x-axis and the line x =1. If the mass per unit area (density) of the sheet is xy find the mass of the sheet.

Solution:



$$\Omega = \{(x,y)|(0 \le x \le 1) and (0 \le y \le x^2)\}$$

Mass:

$$M = \iint\limits_{\Omega} xy\,dx\,dy = \int\limits_0^1 \left(\int\limits_0^{x^2} xy\,dy \right) dx = \int\limits_0^1 x \left(\int\limits_0^{x^2} y\,dy \right) dx = \int\limits_0^1 x * \left(\frac{y^2}{2} \right) \Big|_0^{x^2} dx = \frac{1}{2} \int\limits_0^1 x^5\,dx$$
$$= \frac{1}{12}(x^6)|_0^1 = \frac{1}{12}.$$

Answer: M = $\frac{1}{12}$.